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RESEARCH MEMORANDUN

INVESTIGATION OF A VARIABLE MACH NUMBER SUPERSONIC TUNNEL

WITH NONINTERSECTING CHARACTERISTICS

By John C. Evvard and DeMarquis D. Wyatt

Lewis Flight Propulsion Laboratory Cleveland, Ohio

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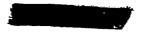
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RESEARCH MEMORANDUM

INVESTIGATION OF A VARIABLE MACH NUMBER SUPERSONIC TUNNEL

WITH NONINTERSECTING CHARACTERISTICS

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SUMMARY

A method is presented for the design of supersonic-wind-tunnel nozzles that produce uniform flow over a continuously variable Mach number range without the use of flexible walls. Experimental results obtained from a preliminary investigation of such a tunnel are included.

Overexpansion of the flow in the neighborhood of the throat was observed at all Mach numbers. Boundary-layer effects were noticeable above the design Mach number of 2.0. Mach numbers were obtained in the range of 1.75 to 2.65 with the limits imposed by either tunnel choking or the structural design of the tunnel. At any given flow-turning angle, less than 1-percent variation in Mach number was observed in the test-section flow downstream of the overexpansion region.

INTRODUCTION

The usual method for obtaining a range of Mach numbers in supersonic wind tunnels is to employ interchangeable fixed nozzle blocks. This technique is subject to the limitations of requiring a large number of nozzles to cover the flow range and to the difficulties of fabricating and maintaining alinement when changes in nozzle blocks are made. Flexible-wall tunnels and sliding non-elastic nozzle contours (reference 1) have been employed to overcome these objections. Solutions to the problem of obtaining theoretically perfect flow in a variable Mach number tunnel have by no means been exhausted and study on the subject is being continued.

The difficulty in designing conventional supersonic tunnels for variable Mach number operation without the use of flexible walls largely arises from problems associated with cancellation of intersecting characteristics originating from the flow-expansion regions. (For the theory of supersonic-wind-tunnel nozzle design, see reference 2.) The calculation of nozzle-wall contours may be simplified if all the expansion waves originate on one wall and are canceled



by the opposite wall, so that intersections of characteristics are thus eliminated. One such tunnel, suggested by Wolfgang Moeckel of the NACA Cleveland laboratory, consists of nozzles having as contours the streamlines generated by Prandtl-Meyer flow around a sharp corner from a Mach number of 1. Construction of an adjustable-angle sharp-expansion corner, although difficult, would produce a variable Mach number supersonic wind tunnel having uniform flow.

Further examination of possible nozzle-wall contours designed with nonintersecting characteristics has revealed that a supersonic tunnel with uniform flow and continuously variable Mach number is achievable without the use of sharp expansion corners or flexible walls. The theory of such a design and the results of a preliminary investigation of a variable Mach number supersonic tunnel based on the theory, which was conducted at the NACA Cleveland laboratory, are presented.

THEORY

Nozzle-wall contours for a supersonic wind tunnel having flow expansion with nonintersecting characteristics are based on the Prandtl-Meyer theory for flow around a corner (reference 3). If an initially uniform stream of sonic velocity is expanded around an arbitrary continuously increasing corner, the Mach number of the flow at the surface depends only on the local angle of flow turning (fig. 1), as given by the equation

$$\theta = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{(\gamma-1)(M^2-1)}{\gamma+1}} + \sin^{-1} \frac{1}{M} - \frac{\pi}{2}$$
 (1)

where θ is the flow-turning angle (the flow-direction angle at that point), γ is the ratio of specific heats, and M is the local Mach number at any point on the surface. The values of Mach number and flow direction generated locally at any point along the expansion surface are constant across the flow field along the Mach line originating at the point. The Mach line forms an angle β with the local surface-tangent line, where

$$\beta = \sin^{-1}\frac{1}{M} \tag{2}$$

By means of equations (1) and (2), the direction field (that is, the system of tangents) of the streamlines may be plotted as shown for a few representative turning angles in figure 1. Graphical



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connection of any set of direction-field lines (for example, line aat of fig. 1) gives a streamline. Any pair of streamlines, including the expansion surface, may be used as nozzle-wall contours in the design of supersonic wind tunnels, and uniform flow results in the test section downstream of the final-expansion Mach line.

As an alternative to the graphical construction of the streamlines, an analytical method may be employed. In order to satisfy one-dimensional isentropic-flow considerations, the area projected normal to the local expansion surface between any two streamlines must equal the one-dimensional-flow area corresponding to the local Mach number at the surface. In terms of the radial distance between streamlines measured along the local Mach line (fig. 1), this relation becomes

$$\mathbf{r} \sin \beta = \mathbf{r}_0 \frac{1}{M} \left(\frac{1 + \frac{\gamma - 1}{2} M^2}{\frac{\gamma + 1}{2}} \right)^{\frac{\gamma + 1}{2(\gamma - 1)}}$$
(3)

or

$$\frac{\mathbf{r}}{\mathbf{r}_0} = \left(\frac{1 + \frac{\gamma - 1}{2} \mathbf{M}^2}{\frac{\gamma + 1}{2}}\right)^{\frac{\gamma + 1}{2(\gamma - 1)}} \tag{5a}$$

where r is the radial distance between streamlines at Mach number M and r_0 is the distance between the same set of streamlines at M = 1.0.

The coordinates \mathbf{x}_1 and \mathbf{y}_1 of the point of intersection of the Mach line with the straightening wall (where the origin of the coordinates is taken at the point on the expansion wall from which the Mach line originates) become simply

$$x_{1} = r \cos (\beta - \theta)$$

$$y_{1} = r \sin (\beta - \theta)$$
(4)

or



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$$\frac{x_{1}}{r_{0}} = \frac{r}{r_{0}} \cos (\beta - \theta)$$

$$\frac{y_{1}}{r_{0}} = \frac{r}{r_{0}} \sin (\beta - \theta)$$
(4a)

Numerical values of β , θ , r/r_0 , x_1/r_0 , and y_1/r_0 computed from equations (1), (2), (3a), and (4a) for a value of $\gamma = 1.400$ are presented in table I as a function of Mach number.

The contour of the flow-straightening wall (aa', fig. 1) depends only on the contour of the expansion surface. The test-section Mach number, on the other hand, depends only on the total angle of flow deflection. If, therefore, the expansion surface is a circular cylinder followed by an attached tangential plate, as in figure 2, the test-section Mach number can be varied by relative rotation of the expansion and straightening surfaces about the axis of the cylinder.

The coordinates X and Y (fig. 2) of the straightening wall relative to the axis of the expansion cylinder may be computed from the equations

$$\frac{X}{r_0} = \frac{x_1}{r_0} + \frac{x_2}{r_0}$$

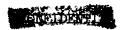
$$\frac{Y}{r_0} = \frac{y_1}{r_0} + \frac{y_2}{r_0}$$
(5)

where x_1/r_0 and y_1/r_0 are obtained from equation (4a). The coordinates of the expansion cylinder x_2 and y_2 are determined from

$$\frac{\mathbf{x}_{2}}{\mathbf{r}_{0}} = \frac{\mathbf{R}}{\mathbf{r}_{0}} \sin \theta$$

$$\frac{\mathbf{y}_{2}}{\mathbf{r}_{0}} = \frac{\mathbf{R}}{\mathbf{r}_{0}} \cos \theta$$
(6)

where R is the radius of the expansion cylinder.

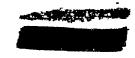


At the Mach number for which the straightening wall is designed, the test section is bounded by the final-expansion Mach line, the tangent plate to the expansion cylinder, and a Mach line originating at the end of curvature of the straightening wall (fig. 2). (The straightening wall is assumed to be terminated at this point.) The resultant test section is triangular. For operation at Mach numbers below the design value (fig. 3(a)), the same restrictions govern the test section except that the final-expansion Mach line intercepts the straightening wall at some point on the curved section. The flow therefore undergoes a compressive turning downstream of this point, the compression region being bounded by a Mach line. The lower limit of operation of the tunnel occurs when compression downstream of the test section causes choking in the passage between the straightening wall and the tangential plate.

For tunnel operation at Mach numbers greater than the design value (fig. 3(b)), a flow expansion occurs in the region between the expansion Mach line corresponding to the design Mach number and the final-expansion Mach line. The expansion region is bounded by a Mach line so that the effective test section is again triangular.

The theoretical maximum test-section lengths over a range of Mach numbers of a variable Mach number tunnel of the type shown in figures 2 and 3 are compared in figure 4 with the test-section lengths of conventional symmetrical-expansion tunnels having either constant test-section heights or constant mass flows. In order to make the results nondimensional, the calculated test-section lengths were divided by the test-section heights at M = 2.0. The variable Mach number tunnel was assumed to be designed for M = 2.0 with operation between M = 1.5 and M = 2.5 and to have a cylinder-radius to throat-height ratio R/r_0 of 1.0. The maximum test-section length for all three tunnels was assumed to be the distance required for a Mach line originating on the center line of the test section to be reflected back to the center line. The center line of the variable Mach number tunnel with nonintersecting characteristics was taken at one-half the maximum test-section height at each condition.

In conventional symmetrical-expansion tunnels having fixed test-section dimensions, the test-section length-height ratio varies only as the cotangent of the Mach angle. The test-section height of conventional tunnels with constant mass flow, on the other hand, so varies with Mach number that the length-height ratio of figure 4 is less than for the fixed-test-section tunnel at Mach numbers below 2.0 and greater for Mach numbers above 2.0.



The variable Mach number tunnel has the same test-section lengths as the conventional constant-mass-flow tunnel at Mach numbers below the design point because the test-section heights are governed by the same considerations. At Mach numbers above the design point, however, the test-section height does not increase in the same manner because of the overexpansion of flow illustrated in figure 3(b). Graphical application of the method of characteristics was used to determine the point of intersection of the finalexpansion Mach line and the Mach line bounding the overexpansion region, and hence the test-section height, for these conditions. The graphical analysis disclosed that the maximum test-section height, and consequently the length-height parameter, approximated that obtained for a conventional fixed test-section tunnel in this range of operation. The position of the model in the tunnel must, of course, be varied in order to attain the maximum length-height ratios of figure 4 for the variable Mach number tunnel. (For many investigations, maximum test-section length is unimportant and a fixed location of the model is possible.)

APPARATUS

Photographs of the variable Mach number tunnel used for these preliminary investigations are shown in figure 5. This tunnel operates on a single-pass, continuous, suction-flow cycle. The subsonic entrance nozzle (at the top of the photograph) is connected by ducting to a 16-inch-diameter pipe supplying air at atmospheric pressure from an activated-alumina air drier. The downstream end of the tunnel was connected to a constant-displacement, reciprocating exhaust pump.

The tunnel is two-dimensional from the beginning of the subsonic entrance nozzle through the test section, with a constant width of 4 inches. A Mach number of 2.0 was arbitrarily chosen as the design Mach number at which complete cancellation of the expansion characteristics would occur. A test-section height of 4 inches was selected for this condition; the nozzle-throat height therefore became fixed at 2.37 inches.

The expansion surface was formed by a cylinder of 4-inch diameter. This cylinder was tangent to a flat plate 7 inches long . that formed one wall of the test section. The tangential plate and the cylinder were rotated to give a wide range of flow turning angles by a manually operated screw-jack mechanism extending outside of the tunnel.





Static-pressure orifices of 0.0135-inch diameter were located on the center line of the fixed-contour wall at 1/2-inch intervals. Similar orifices were staggered at 1/4-inch intervals on lines 1/2 inch to either side of the tangential-plate center line, extending from a point $2\frac{3}{8}$ inches from the point of tangency of the plate and the cylinder to the end of the plate.

A total-pressure survey was made along the center of the tunnel with the pitot pressure rake shown attached to the tangential plate in figure 5. In this photograph the rake is in the position farthest downstream; the rake-attachment design allowed systematic measurements from a point $2\frac{5}{8}$ inches from the point of tangency of the flat plate and the cylinder to the position shown. The pitot tubes used in the survey rake were formed of plugged 1/8-inchdiameter tubes with 0.0135-inch orifices in the ends. The tubes were located at 1/2-inch increments starting 3/8 inch from the flat plate.

A total-pressure tube was installed just upstream of the subsonic entrance nozzle in the 16-inch-diameter pipe to measure the total pressure of the air entering the tunnel. All pressures were measured on a multiple-tube mercury manometer board and were read to the nearest 0.02 inch of mercury.

RESULTS AND DISCUSSION

The results of the calibration survey of the flow in the variable Mach number tunnel are presented in the form of Mach number distributions along the nozzle walls and in the tunnel air stream. Distribution along the tunnel walls was computed from

$$\frac{P_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$
 (7)

where P_0 is the total pressure of the entering air stream measured in the transition section ahead of the entrance nozzle, p is the static pressure measured by a flush wall orifice, γ is the ratio of specific heats (taken as 1.400) and M is the desired Mach number.

Mach number distributions in the tunnel air stream were calculated from the value of the free-stream total pressure measured in



the entrance transition section and the value of the total pressure behind a normal shock as measured with the pitot survey rake by means of the normal-shock total-pressure-ratio equation

$$\frac{P}{P_0} = \left(\frac{\gamma_{+1}}{2\gamma M^2 - \gamma + 1}\right)^{\frac{1}{\gamma - 1}} \left[\frac{(\gamma + 1)M^2}{2\left(1 + \frac{\gamma - 1}{2}M^2\right)}\right]^{\frac{\gamma}{\gamma - 1}}$$
(8)

where P is the total pressure measured by the pitot tube.

Mach numbers along the surface of the straightening wall of the nozzle are presented in figure 6 for the maximum flow_turning angle θ of 43.55°. The experimentally determined Mach numbers show excellent agreement with the theoretically predicted results. The maximum deviation between theoretical and experimental values was about 1 percent. Abrupt discontinuities indicative of strong shock formations were not apparent in the experimental distribution and the small deviations from theory that did occur were probably due to machining irregularities in the wall surface. Agreement between theory and experiment through the full length of the expansion region indicates that isentropic flow existed with negligible boundary-layer growth along this wall. A low boundary-layer growth rate was anticipated because of the favorable pressure gradient.

The Mach number distributions along the surface of the tangential plate are presented in figure 7 for a range of flow-turning angles. These data were obtained without the pitot survey rake in the tunnel, but the minimum turning angle shown ($\theta = 19.65^{\circ}$) corresponded to the lowest Mach number at which the tunnel could be operated without choking when the rake was in position. The turning angle $\theta = 43.55^{\circ}$ was the maximum possible owing to structural interferences.

At each turning angle investigated, the Mach number distribution was slightly irregular. The variations were small, however, with no evidence of strong shock impingement. Shock waves were not visible in the test section by schlieren observation of the flow.

Because theoretically no pressure gradient existed on the tangential plate, the rate of boundary-layer growth was expected to be low. A negligible boundary-layer growth occurred at Mach numbers below the design Mach number of 2.0, as evidenced by the essentially constant Mach number distribution along the



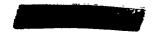


full length of the plate for these conditions. At Mach numbers greater than 2.0, a noticeable boundary layer was formed, with the rate of growth increasing as the Mach number was increased. Because the flow leaving the tangential plate had to undergo a compressive turning of progressively greater angle as the Mach number was increased above 2.0, a positive pressure gradient was probably transmitted back along the surface of the plate to cause the observed thickening of the boundary layer.

Mach number distributions in the tunnel stream as determined from the pitot surveys are shown in figure 8 for seven longitudinal positions of the rake and for several flow-turning angles. Also shown are the Mach numbers determined from static-pressure readings taken simultaneously by wall orifices on the tangential plate directly beneath the pitot-tube tips. The pitot rake extended sufficiently far from the tangential plate that for some rake positions and flow-turning angles one or more of the tubes extended into the expansion region ahead of the final-expansion Mach line. (Inasmuch as the pitot tubes were alined parallel to the tangential plate, an effective angle of attack not exceeding 12° was imposed on the tubes extending into the expansion region.) The theoretical curves shown on figure 8 are therefore constant in value from the tangential plate to the final-expansion Mach line, then decrease as the expansion region is entered.

Reasonable agreement between theoretical and experimental Mach numbers was obtained throughout the region investigated. The maximum discrepancy amounted to less than 3 percent, with the maximum deviation consistently occurring when the pitot tube was near the final-expansion Mach line. The agreement was particularly close in the expansion region, which indicated the validity of assuming isentropic flow in the calculation of the Mach numbers. The experimental Mach numbers were higher in general than the theoretical Mach numbers for the upstream positions of the rake. As the rake was moved downstream in the test section, the experimental Mach numbers decreased below the theoretical values as a result of boundary-layer growth.

The test-section Mach numbers from figure 8 are replotted in figure 9 as a function of the local distance parallel to the tangential plate from each tube to the theoretical final-expansion Mach line. The measurements immediately downstream of the Mach line show good agreement with theory for all flow-turning angles investigated. The high Mach numbers previously noted in figure 8 show a consistent and uniform overexpansion of the flow for a distance of approximately 2 inches downstream of the Mach line.



This overexpansion is attributed to a virtual deformation of the expansion corner either through a boundary-layer protuberance, as predicted for this type of flow by Lees of Princeton, or through flow separation around part of the corner. The effects would be expected to depend on the rate of change of curvature of the expansion surface. Unpublished data from other investigations at the Cleveland laboratory of expansion flow around sharp and rounded corners indicate that the effect does become less pronounced as the strength of the discontinuity in curvature on the expansion surface decreases. The overexpansion region is longest next to the tangential plate and diminishes with distance into the stream, which is indicative of a cancellation of the overexpansion region by compression waves originating at the tangential plate. Such waves were not visible, however, by schlieren observations of the flow.

Downstream of the overexpansion region the flow was reasonably uniform for all flow-turning angles. The experimentally determined Mach numbers in this uniform flow region ranged from 1.75 to 2.65 with less than 1 percent variation in Mach number occurring at any flow-turning angle (fig. 9). The observed Mach numbers were lower than the theoretical values because of boundary-layer effects. The upper Mach number limitations occurred solely because of the structural design of the tunnel and the flow at the maximum Mach number was not accompanied by any effects likely to restrict wider operating ranges in similar tunnels.

SUMMARY OF RESULTS

A preliminary investigation of the flow in a variable Mach number supersonic tunnel with nonintersecting characteristics gave the following results:

1. Overexpansion of the flow in the neighborhood of the throat was observed at all Mach numbers. Downstream of the overexpansion region, test-section Mach numbers were obtained in the range of 1.75 to 2.65 with less than 1 percent variation in Mach number at any given flow-turning angle. The lower and upper Mach number limits were imposed by tunnel choking and structural interference, respectively.





2. Experimental Mach numbers followed theoretical predictions except for boundary-layer effects, which were most noticeable above the design Mach number of 2.0.

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Cleveland, Ohio.

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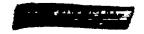


TABLE I. - VARIATION OF NOZZLE DESIGN PARAMETERS

WITH MACH NUMBER FOR $\gamma = 1.400$

For definition of symbols see fig. 1

М	β (deg.)	θ (deg)	r/r ₀	* ₁ /r ₀	y ₁ /r ₀
1.00	90.00	0	1.000	0	1.000
1.10	65.38	1.34	1.109	.486	.997
1.20	56.44	3.56	1.237	.746	.986
1.30	50.29	6.17	1.386	.995	.965
1.40	45.59	8.99	1.561	1.253	.931
1.50	41.81	11.91	1.764	1.529	0.879
1.60	38.68	14.86	2.000	1.830	.808
1.70	36.03	17.81	2.274	2.160	.711
1.80	33.75	20.73	2.590	2.523	.583
1.90	31.76	23.59	2.955	2.925	.420
2.00	30.00	26.38	3.375	3.368	0.213
2.10	28.44	29.10	3.858	3.858	045
2.20	27.04	31.73	4.411	4.396	360
2.30	25.77	34.28	5.044	4.989	747
2.40	24.62	36.75	5.767	5.638	-1.212
2.50	23.58	39.12	6.592	6.351	-1.765
2.60	22.62	41.42	7.530	7.129	-2.427
2.70	21.74	43.62	8.594	7.974	-3.203
2.80	20.93	45.75	9.800	8.896	-4.113
2.90	20.17	47.79	11.164	9.892	-5.176
3.00	19.47	49.76	12.704	10.970	-6.407





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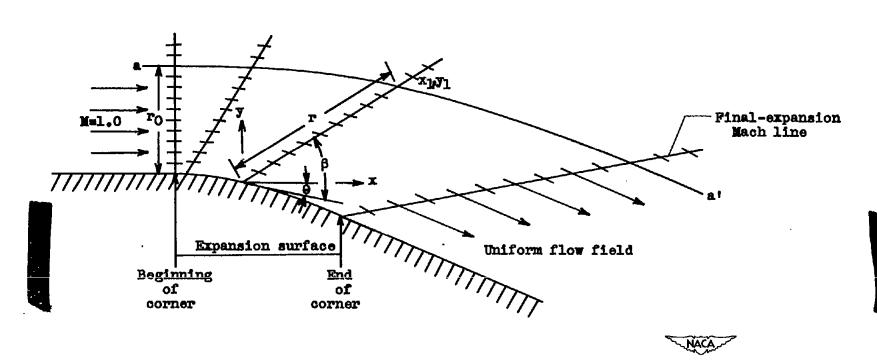
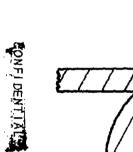


Figure 1. - Flow expansion around a corner.



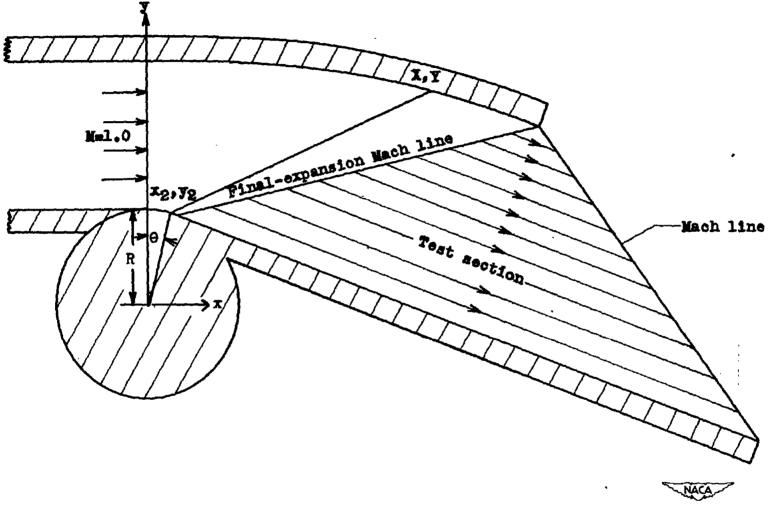
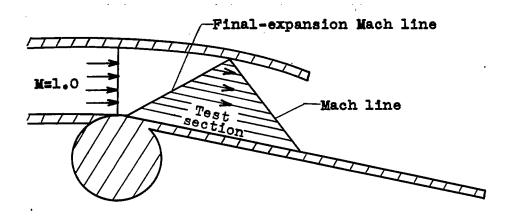


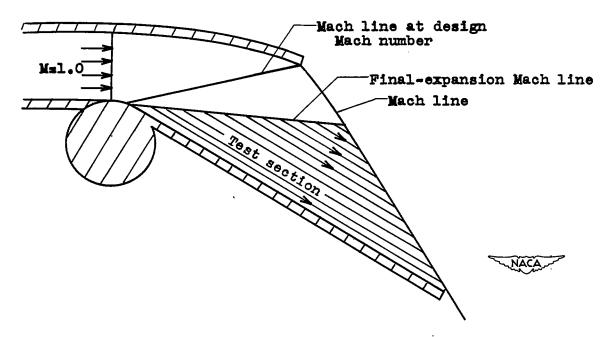
Figure 2. - Schematic drawing of variable Mach number supersonic tunnel with nonintersecting characteristics at design Mach number.

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(a) Test-section Mach number less than design Mach number.



(b) Test-section Mach number greater than design Mach number.

Figure 3. - Test sections resulting from off-design operation of a variable Mach number supersonic tunnel with nonintersecting characteristics.

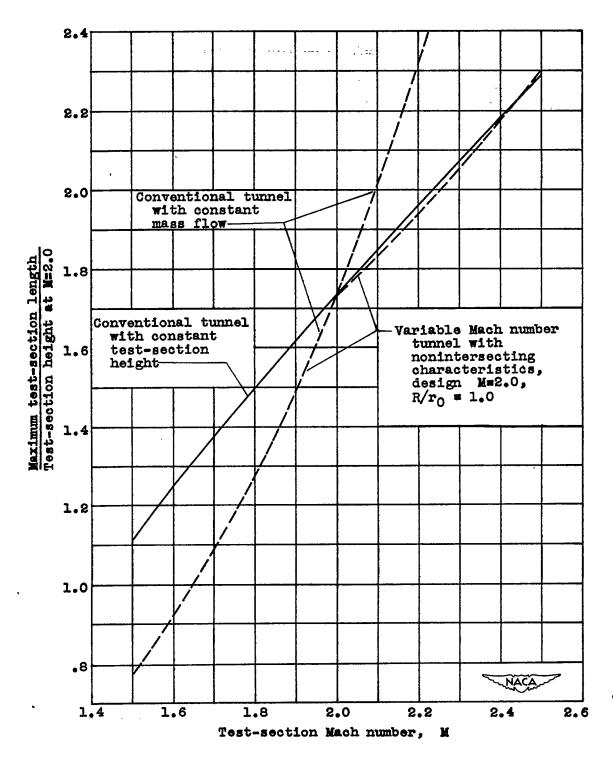
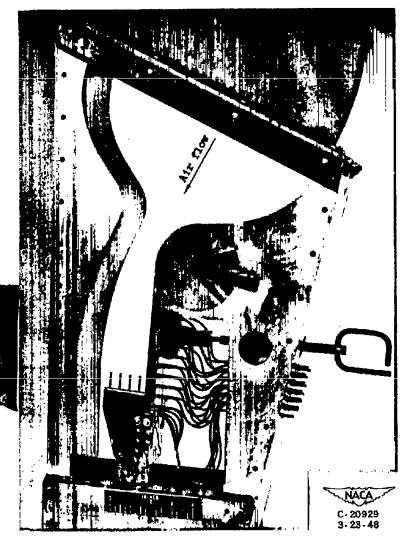


Figure 4. - Comparison of theoretical maximum test-section lengths for several methods of tunnel design.





(a) Right view showing instrumentation from tangent plate and survey rake.



(b) Left view showing instrumentation from straightening wall.

Figure 5. - Variable Mach number tunnel with nonintersecting characteristics.



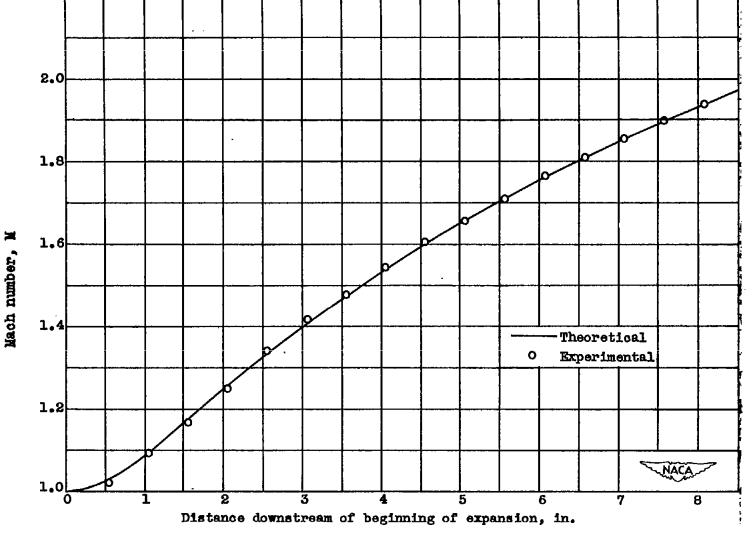


Figure 6. - Comparison of theoretical and experimental Mach number variation along straightening— wall surface. Turning angle θ , 43.55° .

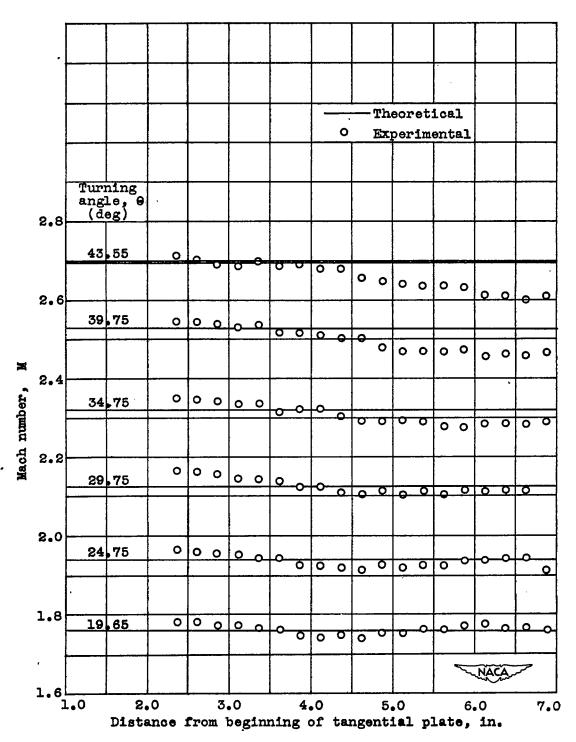
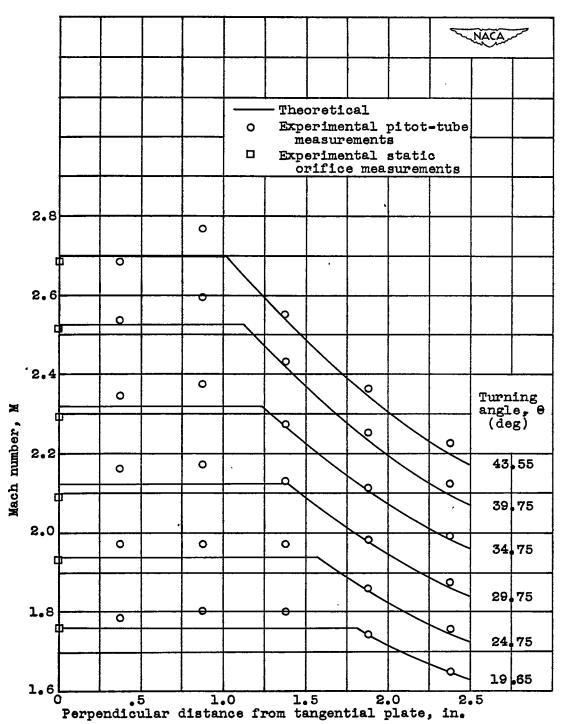


Figure 7. - Comparison of theoretical and experimental Mach numbers determined from static-pressure measurements on tangential plate.

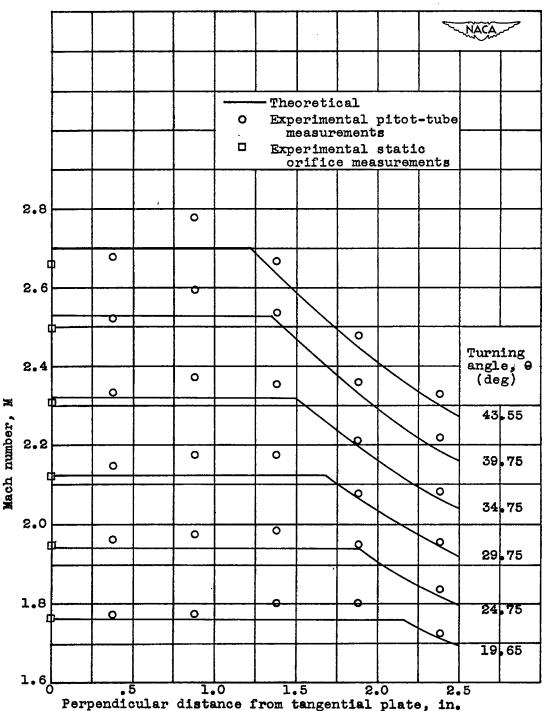




(a) 2.625 inches from beginning of tangential plate.

Figure 8. - Comparison of theoretical and experimental Mach number distribution in vertical plane at center line of tunnel for range of turning angles.



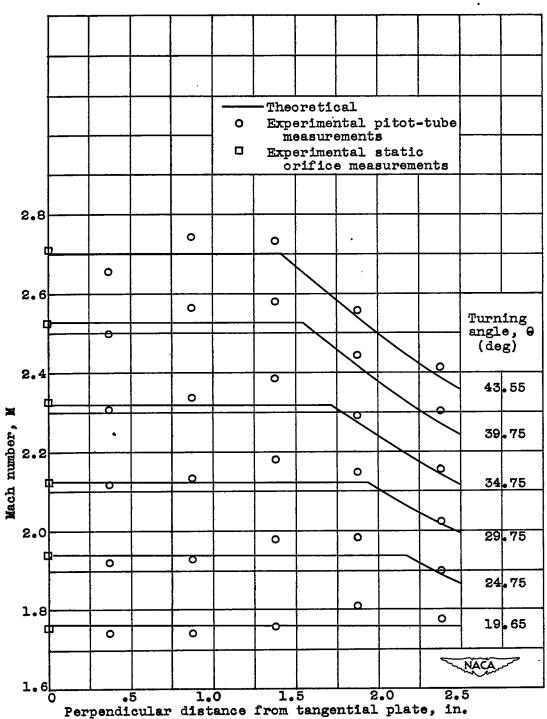


(b) 3.125 inches from beginning of tangential plate.

Figure 8. - Continued. Comparison of theoretical and experimental Mach number distribution in vertical plane at center line of tunnel for range of turning angles.



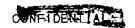


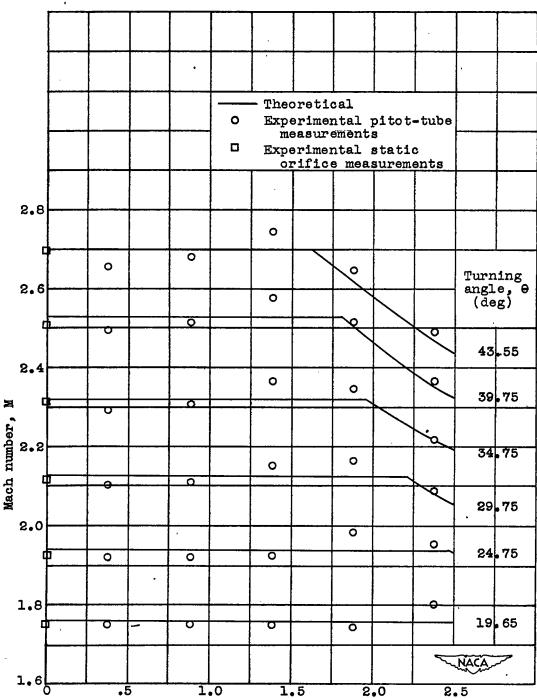


(c) 3.625 inches from beginning of tangential plate.

Figure 8. - Continued. Comparison of theoretical and experimental Mach number distribution in vertical plane at center line of tunnel for range of turning angles.





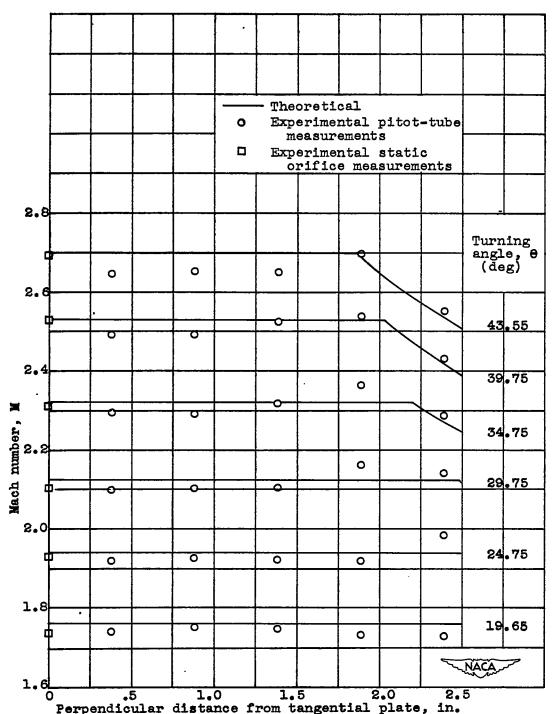


Perpendicular distance from tangential plate, in.

(d) 4.125 inches from beginning of tangential plate.

Figure 8. - Continued. Comparison of theoretical and experimental Mach number distribution in vertical plane at center line of tunnel for range of turning angles.

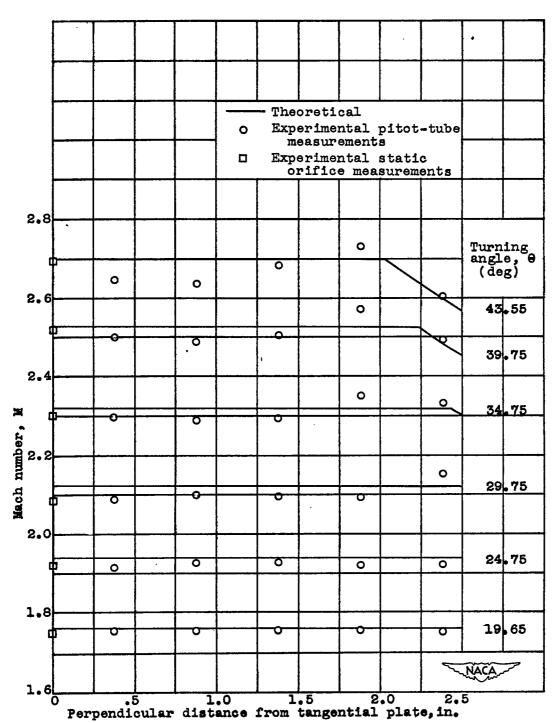




(e) 4.625 inches from beginning of tangential plate.

Figure 8. - Continued. Comparison of theoretical and experimental Mach number distribution in vertical plane at center line of tunnel for range of turning angles.

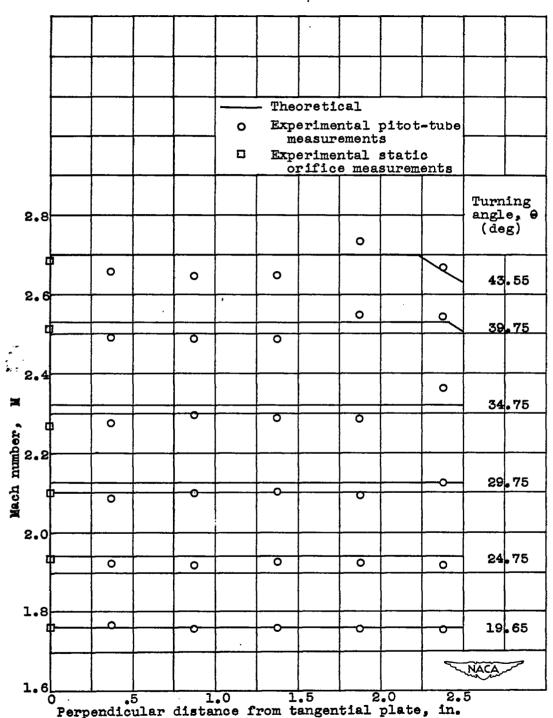




(f) 5.125 inches from beginning of tangential plate. Figure 8. - Continued. Comparison of theoretical and experimental Mach number distribution in vertical plane at center line of tunnel for range of turning angles.







(g) 5.625 inches from beginning of tangential plate.

Figure 8. - Concluded. Comparison of theoretical and experimental Mach number distribution in vertical plane at center line of tunnel for range of turning angles.



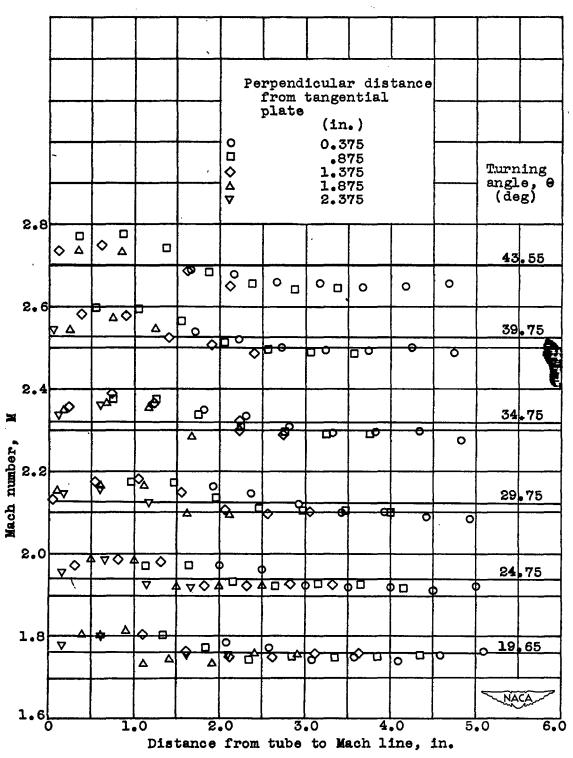


Figure 9. - Variation in Mach number with distance from measuring tube to theoretical final-expansion Mach line.

